Periodic Complementary Sequences for CDMA applications

by

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CERTIFICATE

It is certified that the work contained in the thesis entitled "*Periodic Complementary Sequences for CDMA Applications*" by *Anurag Varshney* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Abstract

Periodic Complementary Sequences(PCS) have good cross correlation properties in the sense that periodic cross correlation is zero for each shift. In this thesis, PCS have been applied to Multicarrier CDMA with same data bits on each carrier. We have compared the BER performance of Multicarrier CDMA using PCS with DS CDMA employing Gold and MLSR sequences in synchronous as well as asynchronous conditions in AWGN channel. We have shown that PCS is quite advantageous in asynchronous conditions and a significant gain is obtained in AWGN. The performance of PCS has also been investigated for Rayleigh fading channel.

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Chapter 1

Introduction

One hundred years ago, the notion of transmitting information without the use of wires must have seemed like magic. Marchese Marconi made it possible. In 1896, the first patent for wireless communications was granted to him in United Kingdom. In fact, he demonstrated the first wireless communications system in 1897 between a land base station and a tug boat. Since then, unbelievable, extraordinary and rapid developments in the field of wireless communications have been taking place that is expected to shrink the world into a global communication village(GCV) by next 20 years. A GCV system will provide communications from one point to other in any place at any time without any delay with acceptable quality and security.

Multiple access schemes play a significant role in wireless communications. By multiple access, we mean that several users can share the same channel simultaneously. Since there are multiple users transmitting over the same channel, a method must be established so that individual users will not disrupt one another. Two such multiple access schemes are Frequency Division Multiple Access(FDMA), Time Division Mul-

tiple Access(TDMA) [11]. The idea behind FDMA is to split up the available channel into non-overlapping frequency bands and each active user will be assigned an individual band. In this case, the receiver can tune to the specified band and demodulate the information. In a TDMA system, the channel is divided into time segments and a time slot is assigned to each active user. In these systems, each users is essentially orthogonal in either time or frequency, which makes detection a relatively easy task.

But the scheme that is gaining huge attention in recent times is Spread Spectrum technology more popularly known as CDMA i.e. Code Division Multiple Access. Spread spectrum technology first found life when it was pioneered by the military because of its high resistance to jamming and secure form of modulation. While in today's world the threat of jamming may not be a primary concern, but certainly security in the digital age in which we live is paramount. Spread spectrum modulation works by allowing each user to utilize the full allotted bandwidth for their communications. Spread spectrum overcomes this problem by associating with each user a unique spreading code which is specific to that user. CDMA scheme of data transmission is a wideband transmission scheme in which the bandwidth of the signal is spread manyfold times than what is required for actually transmitting the signal by multilpying the data bits of user by the spreadng sequence. Thus when it is time to demodulate each user's signal the receiver must have knowledge of the spreading code to extract the signal form all the other signals.

Some of the advantages which a CDMA system offers are:

1) Robust to intereference and jamming.

2) Improved security and privacy.

3) Improved capacity.

4) CDMA's patented "soft handoff," method of passing calls between cells sharply reduces the risk of disruption or dropped calls during a handoff.

5) CDMA technology enables users to access a wide range of new services, including caller identification, short messaging services and Internet connections. Simultaneous voice and data calls are also possible using CDMA technology.

6) CDMA's spread spectrum signal provides the greatest coverage in the wireless industry, allowing networks to be built with far fewer cell sites than is possible with other wireless technologies.

7) CDMA systems can be deployed and expanded faster and more cost effectively than most wireline networks. And because they require fewer cell sites, CDMA networks can be deployed faster than other types of wireless networks.

As mentioned earlier, users in a CDMA system are distinguished by user specific spreading sequences. Whenever data is transmitted over a channel, errors are encountered either due to impairments of the channel or by interference among simultaneous users. The Multi Access Interference(MAI) is a major source of transmissions errors in such scheme and it is due to the non ideal characteristics of applied spreading sequences. It can be mitigated either by utilising very complicated and costly multiuser detection algorithms or simply by reducing the number of simultaneously active users. However none of these approaches seem to be economically viable. In fact, we want to maximize the number of simultaneous users without any significant loss of data relia-

bility. Additionally we want to minimize the cost of transceivers in order to broaden the scope of possible users. Therefore, design of signature sequences and their utilization have become a fundamental and important problem and need to be addressed. Moreover, it is expected that in the radio interface, there will exist data services characterized by different data rates. In order to accomodate such services, and provide for optimal bandwidth utilization, the radio signal in all cases should occupy the same bandwidth. Therefore, the sequences should be of any arbitary length when required. Also, we know that transmissions in uplink are normally asynchronous. Therefore, it is advantageous that sequences should be orthogonal for the complete elimination of MAI even in asynchronous environment.

While considering signature sequences for different users, the most important property is the cross correlation betwen the sequences. The cross correlation between the sequences should be as small as possible to ensure the complete elimination of MAI. Ideally it should be zero, i.e. the spreading codes should be completely orthogonal to each other. Also, the capacity of CDMA system depends upon the number of sequences possible for a given length of sequence. More is the number of sequences, more is the number of users that can be accomodated in the system.

1.1 Motivation For Present Work

There has been an enormous research in field of CDMA in past few years. The most widely used sequences in CDMA like Walsh sequences are orthogonal to each other but orthogonality is completely destroyed in asynchronous environment. But the uplink in

wireless is asynchronous. Thus, there can be a major interference among users leading to errors in transmission. Hence, serious attempt has been made to preserve the orthogonality between different users to minimize MAI even in asynchronous environment. From this point onwards, asynchronous will mean that delay between users is an integer multiple of chip duration.

1.2 Organisation of Thesis

The thesis is organised as follows: In chapter 2, we discuss the basic principles and architecture of DS CDMA system and multicarrier CDMA. The properties of spreading sequences (Gold, MLSR, Kasami, Walsh) and their method of construction is also given. Chapter 3 deals with periodic complementary sets of sequences in detail. Multicarrier CDMA architecture for transmitting PCS is shown. Methods are presented to construct PCS of varying length and to increase the number of sequences in a particular set.

Chapter 4 gives the simulation results and conclusions. The performance of PCS is compared with Gold and MLSR sequences in AWGN channel. Both synchronous and asynchronous systems have been considered. Rayleigh fading (slow and flat) is also considered for simulations.

Chapter 2

Spread Spectrum Systems

2.1 Introduction

In Code Division Multiple Access (CDMA) system [12], all users transmit in the same bandwidth simultaneously. Communication systems following this concept are "Spread Spectrum Systems". In this transmission technique, the frequency spectrum of a data-signal is spread as a result of which the bandwidth occupancy is much higher then required. There exist different techniques to spread a signal namely Direct-Sequence (DS), Frequency-Hopping (FH), Time-Hopping (TH) and Multi-Carrier CDMA. It is also possible to make use of combinations of these techniques.

2.2 DS CDMA System

In DS CDMA system, the narrow band data is spread by multiplying it with a spreading code. The codes used for spreading have low cross correlation values and are unique to every user. This is the reason that a receiver which has knowledge about the code of the intended transmitter, is capable of selecting the desired signal. The rate of

the code known as chip rate is thus higher than the data rate of the user. The main parameter in spread spectrum systems is the processing gain, the ratio of transmission and information bandwidth.

$$G_p = \frac{BW_t}{BW_i} = \frac{R_c}{R_b} = \frac{T_b}{T_c} \tag{2.1}$$

The processing gain determines the number of users that can be allowed in a system, the amount of multi-path effect reduction, the difficulty to jam or detect a signal etc. For Spread Spectrum System it is advantageous to have a processing gain as high as possible. A simple diagram of DS CDMA is shown in Figure 2.1.

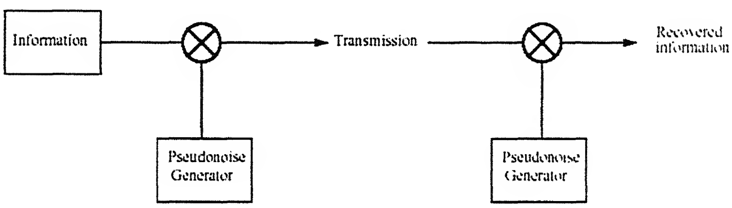

Figure 2.1: A simplified DS CDMA system

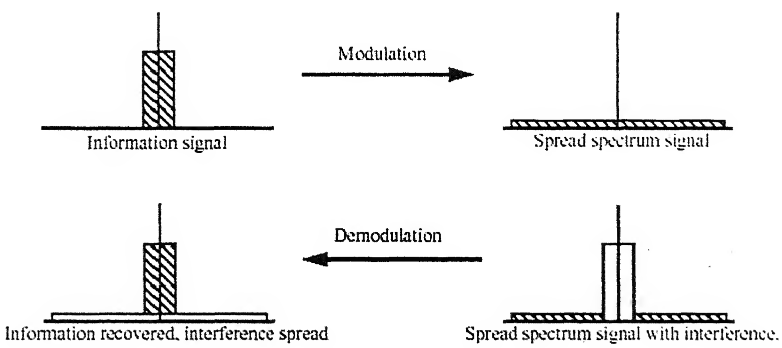

Figure 2.2: Effect of Spreading and Despreading

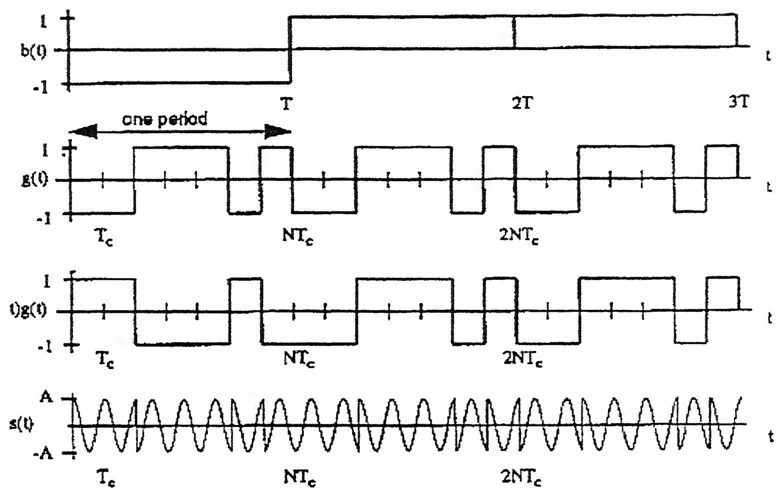

Figure 2.3: Different waveforms a)Bit sequence b)Chip sequence c)Multiplied sequence d)Transmitted modulated waveform

The signal is usually recovered at the receiver by correlating the incoming spread signal with the same unique sequence that is used to encode the signal. This despreads the received signal and returns it to original bandwidth at the receiver. However, any narrow band intereference present in spread-width of the information signal will be spread at the receiver and can easily be eliminated. This is the narrow band rejection property of CDMA. Other advantages of CDMA are givcn the first chapter.

2.3 Synchronous CDMA Model

In synchronous CDMA model [5], bit epochs are aligned at the receiver. This requires closed loop timing control or providing the transmitters with access to a common clock. In cellular systems, the downlink (base to mobile station) is synchronous.

The basic synchronous CDMA model of K users embedded in additive white Gaus-

sian noise can be defined as

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t) \qquad , t \in [0, t] \tag{2.2}$$

Here,

T is inverse of data rate.

$s_k(t)$ is the deterministic signature waveform assigned to kth user.

A_k is the received amplitude of kth user's signal.

$b_k \in \{+1, -1\}$, is the bit transmitted by kth user.

2.4 Asynchronous CDMA Model

In case of asynchronous CDMA [5], different users transmit data independent of each other and there is no synchronism between the bit epochs at the receiver. The uplink in a wireless system is usually asynchronous. Assuming a K user system again, where kth user transmits a packet of 2M+1 bits denoted by

$$\{b_k(-M), ..., b_k(0), ..., b_k(M)\}$$

The asynchronous CDMA model can be written as

$$y(t) = \sum_{k=1}^{K} \sum_{-M}^{M} A_k b_k(i) s_k(t - iT - \tau_k) + \sigma n(t) \tag{2.3}$$

Here, τ_k is the delay of kth user. It is defined with repect to some fixed origin. If T is the bit interval, $\tau_k \in [0, T]$. If N is the processing gain and integer delays are taken into account, then the number of delays possible is N-1. Synchronous CDMA is special case of asynchronous CDMA with

$$\tau_1 = \tau_2 = ... = \tau_K$$

2.5 Multicarrier CDMA

In Multicarrier CDMA system, a data sequence multiplied by a spreading sequence modulates M carriers, rather than a single carrier [3]. The receiver provides a correlator for each carrier, and the outputs of the correlators are combined to yield a processing gain comparable to that of a single carrier DS system. This type of system has the following advantages. First, a Multicarrier system is robust to multipath fading. Second, a Multicarrier CDMA system has a narrowband interference suppression effect. Third, a lower chip rate is required, since, in a Multicarrier system with M carriers, the entire bandwidth of the system is divided into M equi-width frequency bands, and thus each carrier frequency is modulated by a spreading sequence with a chip duration which is M times as long as that of a single carrier system. In other words, a Multicarrier system requires a lower speed, parallel-type of signal processing, in contrast to a fast, serial-type of signal processing in a single carrier RAKE receiver. This, in turn, might be helpful for use with a low power consumption device. Finally, a Multicarrier system does not require a contiguous frequency band, so that very large spread bandwidths are achievable.

Figure 2.4 shows the frequency sprectrum of DS CDMA and Multicarrier CDMA

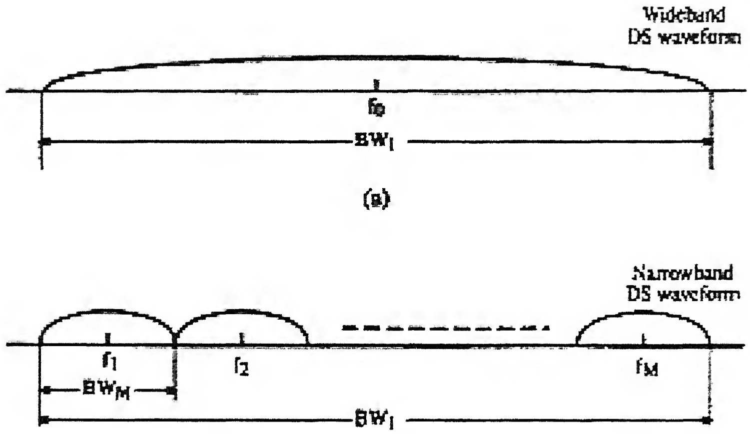

Figure 2.4: Spectrum of a) DS CDMA b) Multicarrier CDMA

The Multicarrier CDMA systems are basically categorized into two groups. The first spreads the original data stream using a given spreading code and then modulates a different subcarrier with each chip. In one way, we can say that spreading is in frequency domain [3]. The other spreads the serial to parallel converted data streams using a given spreading code, and then modulates a different sub carrier with each of the data stream. Thus, the spreading here is in time domain like DS CDMA system. In all the cases above, different data is transmitted on different carriers.

There are three types of multicarrier schemes given as-

1)MC CDMA

2)multicarrier DS CDMA

3)MT-CDMA

Out of these, MC CDMA lies in first category and remaining two in second category.

2.5.1 MC-CDMA

MC-CDMA transmitter spreads the original signal using a given spreading code in the frequency domain. In other words, a fraction of symbol corresponding to a chip of the spreading code is transmitted through a different subcarrier. For multicarrier transmission, it is essential to have frequency non selective fading over each carrier. Therefore, if the original symbol rate is high enough, the signal first needs to be serial to parallel converted before spreading in frequency domain.

2.5.2 Multicarrier DS CDMA Schemes

Multicarrier DS CDMA transmitter spreads the serial to parallel(S/P) converted data using a given spreading code in time domain so that resulting spectrum of each subcarrier satisfies the orthogonality condition with minimum frequency separation. This scheme can lower the data rate in each subcarrier so that a large chip time makes it easy to synchronize the spreading sequences.

2.5.3 MT-CDMA Scheme

The MT-CDMA transmitter spreads the S/P converted data in time domain so that the spectrum of each subcarrier, before spreading operation can satisfy the orthogonality condition with minimum frequency separation. Therefore, the resulting spectrum of each subcarrier no longer satisfies the orthogonality condition. The MT-CDMA scheme uses longer spreading code than number of subcarriers. The MT-CDMA scheme suffers from intersubcarrier intereference, while the capability to use longer spreadig codes

results in reduction of self intereference and MAI, as compared with the spreading codes assigned to a normal DS CDMA scheme.

Transmitter and Receiver structures of above types of multicarrier schemes are given in [3].

2.6 Spreading Sequences

Four basic classes of code sequences are widely used for CDMA applications. They include PN, Gold, Kasami, and Walsh sequences. Apart from this, some other spreading sequences like Legendre and Twin-prime sequences [9] are also used. We will discuss the first four types of spreading sequences in this section.

2.6.1 Pseudorandom Sequences and MLSR Sequences

Pseudorandom Noise Sequences or PN Sequences [10] are known sequences which exhibit the properties or characteristics of random binary sequences. A Pseudorandom Noise (PN) sequence is a sequence of binary numbers, which appears to be random but is in fact perfectly deterministic.

These PN sequences are generated using linear binary shift registers. Figure 2.5 illustrates this.

The maximum length of a PN sequence is determined by the length of the register and the configuration of the feedback network. An N bits register can take up to 2^N different combinations of zeros and ones. Since the feedback network performs linear operations, if all the inputs (i.e. the content of the flip-flops) are zero, the output of

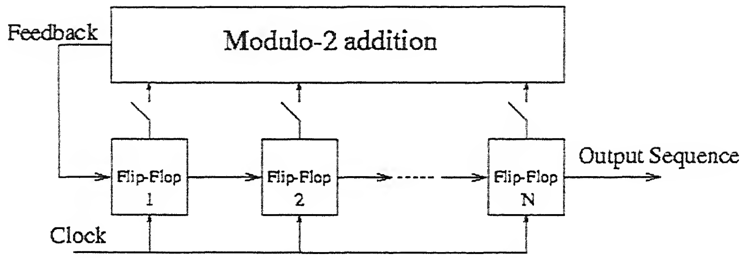

Figure 2.5: Shift Register for generation of PN sequences

the feedback network will also be zero. Therefore, the all zero combination will always give zero output for all subsequent clock cycles and so we do not include it in the sequence. Thus, the maximum length of any PN sequence is $2^N - 1$ and sequences of that length are also called Maximal-Length Sequences or m-sequences or MLSR sequences. The flip-flops which should be tapped-off and fed into the module 2 adder are determined by an advanced algebra which has identified certain binary polynomials called primitive irreducible or unfactorable polynomials. Such polynomials are used to specify the feedback taps. For example, IS-95 specifies the in-phase PN generator shall be built based on the characteristic polynomial

$$P(x) = x^{15} + x^{13} + x^9 + x^8 + x^7 + x^5 + 1$$

The MLSR sequences satisfy the following properties [11]-

1)The number of ones is always one more than the number of zeros in each period.

2)Among the runs of ones and zeros in each period, half of the runs are of length 1, one fourth are of length two, one eigth are of length 3 and so on.

3)The autocorrelation of a MLSR sequence closly resembles that of a random binary wave. It is periodic and binary valued. Let c(t) denote the waveform of the MLSR

sequence. The signal c(t) is periodic with period $T_b = NT_c$. The autocorrelation function of c(t) is given by

$$R_c(\tau) = \frac{1}{T_b}\int_{T_b/2}^{T_b/2} c(t)c(t-\tau)dt \tag{2.4}$$

From this formula, we obtain

$$R_c(\tau) = 1 - \frac{N+1}{NT_c}|\tau| \quad |\tau| \leq T_c \tag{2.5}$$

$$R_c(\tau) = -1/N, \quad \text{elsewhere} \tag{2.6}$$

The autocorrelation function of MLSR sequences is shown in Figure 2.6.

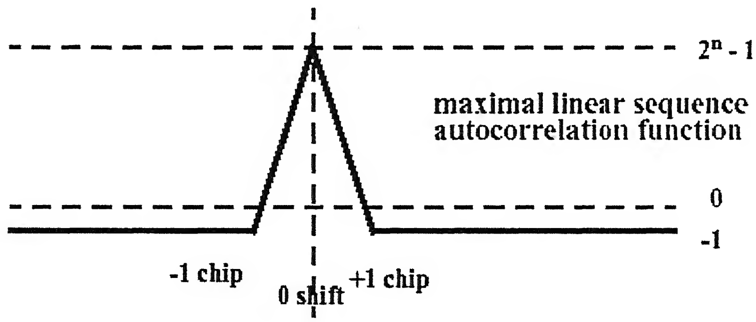

Figure 2.6: Autocorrelation waveform for Pseudorandom sequence.Only one period is shown.

One important property of MLSR sequence is that if we shift a MLSR sequence and do bitwise xor with the original sequence, we get a new MLSR sequence [7]. This property and the first property explained above is responsible for excellent autocor-

relation properties of MLSR sequences. However, the cross correlation properties are poor.

2.6.2 Gold Sequences

Since the cross correlation property of MLSR sequences is not good, they are not suitable for CDMA applications. To have good cross correlation properties, we use another class of PN sequences called Gold Sequences [13].

For generating Gold codes, two MLSR sequences are used. All MLSR sequences do not generate Gold sequences. Only some pairs of MLSR sequences generate Gold sequences and are known as preferred pairs. In order to generate Gold sequences we generate two preferred MLSR sequences. Suppose **a** and **b** are 2 sequences of length N. Then a set of sequences is constructed by taking the modulo 2 sum of **a** with N cyclically shifted versions of **b** or vice versa. Thus for length N, we get N+2 Gold sequences. The set G(**a**, **b**) of Gold sequences is defined by

$$G(\mathbf{a},\mathbf{b}) = \{\mathbf{a},\mathbf{b},\mathbf{a}\oplus\mathbf{b},\mathbf{a}\oplus T\mathbf{b},...,\mathbf{a}\oplus T^{N-1}\mathbf{b}\} \tag{2.7}$$

where T represents the operator that shifts vectors cyclically to the left by one place, and $\oplus$ represents addition modulo 2.

Gold sequences have three valued auto and cross correlation function with values given by $(-1, -t(m), t(m)-2)$ where

$$t(m) = 2^{m+2/2} + 1 \quad \text{for even m} \tag{2.8}$$

$$t(m) = 2^{m+1/2} + 1 \quad \text{for odd m} \tag{2.9}$$

Figure 2.7 shows the generation of Gold sequences.

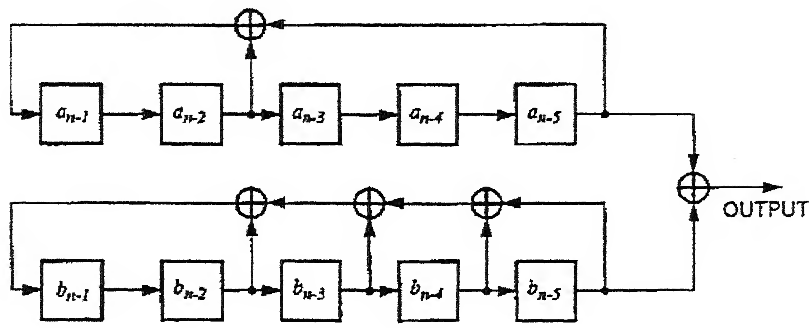

Figure 2.7: Construction of Gold Sequences of Length 31

2.6.3 Kasami Sequences

Kasami sequences have optimal cross correlation values touching the Welch lower bound. A lower bound on the cross correlation between any pair of binary sequences of period n in a set of M sequence is

$$\Phi_{max} \geq n\sqrt{\frac{M-1}{Mn-1}} \tag{2.10}$$

To generate a set of Kasami sequences, begin with an MLSR sequence **a** with period $P = 2^r - 1$ where r is even. Form a binary sequence **b** by taking every bit of **a**. Sequence **b** is periodic with period $2^{r/2} - 1$. By repeating **b** $2^{r/2} + 1$ times , we obtain sequence **c** of length $2^r - 1$. We form a new set of sequences by adding modulo 2 the bits of **a** and **c** and all $2^{r/2} - 2$ cyclic shifts of **c**. Thus the total number of sequences is $2^{r/2}$. The cross correlation and auto correlation take the values from the set $\{-1, -(2^{r/2}+1), 2^{r/2}-1\}$. The maximum cross correlation of the sequences from the set is:

$$\Phi_{max} = 2^{r/2} + 1$$

This value of Φ_{max} satisfies the Welch bound for a set of $2^{r/2}$ sequences of length $2^r - 1$. Hence, the Kasami sequences are optimal.

2.6.4 Walsh Sequences

The Walsh functions or sequences of order N is defined as a set of N time functions denoted as

$$\{W_j(t), t \in (0, t), j = 0, 1, ..., N-1\}$$

$W_j(t)$ takes on the values +1,-1 except at the jumps where its value is 0. The Walsh function $W_j(t)$ satisfies the following properties:

1) $W_j(0) = 1 \ \forall j$

2) $W_j(t)$ has precisely j sign changes in the interval (0,t).

3) The Walsh functions are orthogonal to each other.

$$\int_0^T W_j(t) W_k(t) dt = 0 \quad if\ j \neq k \quad and\ T\ if\ j = k$$

However, the autocorrelation function of Walsh-Hadamard sequences does not have good characteristics. It can have more than one peak and therefore, it is not possible for the receiver to detect the beginning of the sequence without an external synchronization scheme. The cross correlation can also be non zero for a number of time shifts and unsynchronized users can interfere with each other. This is why Walsh-Hadamard sequences can only be used in synchronous CDMA.

The Walsh codes can be generated using Hadamard matrices [12]. The Hadamard matrix is a square array of plus and minus ones, whose rows and columns are mutually orhogonal to each other. The Hadamard matrix of order N is denoted by H_N. The

Hadamard matrix of order 2N is recursively generated from the formula

$$H_{2N} = \begin{pmatrix} H_N & H_N \\ H_N & -H_N \end{pmatrix}$$

where $H_1 = [1]$ The rows of the Hadamard matrix is the set of Walsh sequences. Walsh-Hadamard sequences are important because they form the basis for orthogonal sequences with different spreading factors. This property becomes useful when we want signals with different spreading factors to share the same frequency channel. The sequences that posses this property are called Orthogonal Variable Spreading Factor (OVSF) sequences.

Chapter 3

Periodic Complementary Sequences

3.1 Introduction

Binary complementary sequences were introduced by Golay in 1949 [1]. They were called as Aperiodic Complementary Sequences(ACS). But the limitation was that ACS involved only two sequences. The ACS were modified by Tseng and Liu in 1972 [2]. They derived ACS with more than two sequences but still only even number of sequences were possible. Binary complementary sequences include one more class of sequences called Periodic Complementary Sequences(PCS). PCS exists for both odd and even number of sequences.

In this chapter, we will discuss Periodic Complementary Sequences in detail. Their autocorrelation and various invariance properties are discussed in the subsequent sections. Methods are given to construct PCS of longer periods using PCS of shorter periods. Finally, we propose a multicarrier CDMA scheme using PCS as the spreading sequences. We denote a spreading sequence by boldface letters.

3.1.1 Definition

Consider a binary sequence of length N given by

$\mathbf{a} = (a(0), a(1), \ldots a(N-1))$

Here a(i)= +1 or -1 ,$0 \leq i \leq N-1$

The aperiodic and periodic autocorrelation functions of $\mathbf{a}$ are defined as [6]

$$A_{\mathbf{a}}(k) = \sum_{j=0}^{N-1-k} a(j)a(j+k), \quad 0 \leq k \leq N-1 \tag{3.1}$$

$$P_{\mathbf{a}}(k) = \sum_{j=0}^{N-1} a(j)a(j+k), \quad \text{(j+k)modN}, 0 \leq k \leq N-1 \tag{3.2}$$

Let $\{\mathbf{a}_1, \mathbf{a}_{2,\ldots\ldots\ldots}\mathbf{a}_m\}$ be a set of binary sequences with m elements and each sequence be of length N. Then this set is called a set of Aperiodic Complementary Sequences(ACS) if sum of their aperiodic autocorrelation functions is zero for all shifts excluding the zero shift. In equation form, it may be shown as

$$\sum_{i=1}^{m} A_{\mathbf{a}_i}(k) = 0, \quad 1 \leq k \leq N-1 \tag{3.3}$$

$$\sum_{i=1}^{m} A_{\mathbf{a}_i}(0) = Nm \tag{3.4}$$

The set is called a set of Periodic Complementary Sequences(PCS) if the sum of periodic autocorrelation functions of the sequences involved is zero for all cyclic shifts except at the zero shift. Mathematically, it can be written as

$$\sum_{i=1}^{m} P_{\mathbf{a}_i}(k) = 0, \quad 1 \leq k \leq N-1 \tag{3.5}$$

$$\sum_{i=1}^{m} P_{\mathbf{a}_i}(0) = Nm. \tag{3.6}$$

In other words, we can say that sidelobes of autocorrelation function cancel out completely in case of ACS and PCS which is very difficult in case of traditional CDMA sequences. A PCS may alternatively be defined by the two dimensional periodic autocorrelation function of a binary array. A set of P sequences of length N($\mathbf{a}_i, 0 \leq i \leq P-1$) is called a binary array denoted by

$$(a_{i,j}, 0 \leq i \leq P-1, 0 \leq j \leq N-1)$$

The two dimensional periodic autocorrelation function of the binary array $a_{i,j}$ is defined as

$$\phi_{u,v} = \sum_{i=0}^{P-1} \sum_{j=0}^{N-1} a_{i,j} a_{i+u,j+v} \quad , \text{(i+u)modP and (j+v)modN} \tag{3.7}$$

A binary array $a_{i,j}$ is called perfect if

$$\phi_{u,v} = P.N, \quad \text{u=0modP ,v=0modN} \tag{3.8}$$

$$\phi_{u,v} = 0, \quad \text{elsewhere} \tag{3.9}$$

If u=0, (3.7) reduces to (3.5). Thus the rows of perfect binary array form a PCS. In other words, we can say that if the non zero horizontol shifts of the two dimensional periodic autocorrelation function(PACF) contributes zero, then the rows of array are sequences of PCS. While ACS are limited to even number of sequences, PCS exists for both even number and odd number of sequences.

For the sake of convention, a periodic complementary set of P binary sequences of length N will be denoted by S_P^N. The position of particular bit in a sequence will be indicated by paranthesis and a particular sequence in the set of PCS will be indicated by a subscript.

In practical applications, usually the aperiodic auto and cross correlation functions are used. However, if a set of periodic complementary sequences is transmitted and correlated with twice repeated sequences of the set of PCS, then the sum of resulting aperiodic correlation functions is zero between two main peaks in one period.

Consider an example of PCS given by

$$S_3^8 = \begin{pmatrix} - & + & - & + & + & + & + & + \\ - & - & + & - & + & + & + & + \\ - & - & + & + & - & + & + & + \end{pmatrix} \tag{3.10}$$

Here '-'denotes -1 and '+' denotes +1. The 3 sequences and their corresponding periodic autocorrelation values are-

$$\mathbf{a_1} = (- + - + + + + +) \tag{3.11}$$

$$P_{\mathbf{a_1}} = (8\ 0\ 4\ 0\ 0\ 0\ 4\ 0) \tag{3.12}$$

$$\mathbf{a_2} = (- - + - + + + +) \tag{3.13}$$

$$P_{\mathbf{a_2}} = (8\ 0\ 0\ 0\ -4\ 0\ 0\ 0) \tag{3.14}$$

$$\mathbf{a_3} = (- - + + - + + +) \tag{3.15}$$

$$P_{\mathbf{a_3}} = (8\ 0\ -4\ 0\ 4\ 0\ -4\ 0) \tag{3.16}$$

The sum of three periodic autocorrelation functions is: P=(24 0 0 0 0 0 0 0) which is zero except at zero shift.

3.2 Mates of Periodic Complementary Sequences

Let $\mathbf{A}_i = \{\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_m\}$ be one set of PCS and $\mathbf{B}_i = \{\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m\}$ be another set of PCS. Then $S_m^N(\mathbf{b}_i)$ is called a mate of $S_m^N(\mathbf{a}_i)$ if and only if

$$\sum_{i=1}^{m} \phi_{\mathbf{a}_i\mathbf{b}_i}(k) = 0, \quad 0 \leq k \leq N-1, \tag{3.17}$$

Here $\phi_{\mathbf{a}_i\mathbf{b}_i}(k)$ is cross correlation for kth shift and is defined as

$$\phi_{\mathbf{a}_i\mathbf{b}_i}(k) = \sum_{j=0}^{N-1} a_i(j)b_i(j+k)\ , \quad \text{j+k modN},\ 0 \leq k \leq N-1 \tag{3.18}$$

In other words, we can say that sum of periodic cross correlation of the sequences of two mates involved is zero everywhere including the zero shift. This is a very significant and unique property and can be utilized for CDMA purposes where orthogonality between sequences is of utmost importance. This is the property which makes mutually orthogonal sets to have zero multiaccess interference in AWGN channel.

Following is an example of orthogonal PCS:

Let two PCS be given by

$$\begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \mathbf{a}_4 \end{pmatrix} = \begin{pmatrix} + & + & + & + & + & + & - & - \\ + & + & - & - & - & - & - & - \\ - & + & - & + & - & + & + & - \\ - & + & + & - & + & - & + & - \end{pmatrix} \tag{3.19}$$

$$\begin{pmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \mathbf{b}_4 \end{pmatrix} = \begin{pmatrix} - & + & - & + & - & + & + & - \\ - & + & + & - & + & - & + & - \\ + & + & + & + & + & + & - & - \\ + & + & - & - & - & - & - & - \end{pmatrix} \tag{3.20}$$

The periodic cross correlation between corresponding sequences of two sets is given by-

$$\phi_{a_1b_1}=\{0,-4,0,0,0,0,0,4\} \tag{3.21}$$

$$\phi_{a_2b_2}=\{0,-4,0,0,0.0.0,4\} \tag{3.22}$$

$$\phi_{a_3b_3}=\{0,4,0,0,0,0,0,-4\} \tag{3.23}$$

$$\phi_{a_4b_4}=\{0,4,0,0,0,0,0,-4\} \tag{3.24}$$

Hence, the sum of cross correlation function is

$$\phi_{a_1b_1}+\phi_{a_2b_2}+\phi_{a_3b_3}+\phi_{a_4b_4}=\{0,0,0,0,0,0,0,0\} \tag{3.25}$$

Thus the two PCS are mates of each other. The sum of cross correlation is zero at each and every point. This illustrates the ideal cross correlation property of mates.

The following example shows two PCS which are not mates of each other:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} - & - & + & + & - & + & - & + \\ + & - & - & + & - & + & - & + \\ + & + & - & - & + & + & + & + \\ - & - & - & + & + & - & - & - \end{pmatrix} \tag{3.26}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} - & + & - & + & - & + & + & - \\ - & + & + & - & + & - & + & - \\ + & + & + & + & + & + & - & - \\ + & + & - & - & - & - & - & - \end{pmatrix} \tag{3.27}$$

The periodic cross correlation between corresponding sequences of two sets is given by-

$$\phi_{a_1b_1}=\{0,0,0,0,4,-8,4,0\} \tag{3.28}$$

$$\phi_{\mathbf{a_2b_2}}=\{-8,4,0,0,0,0,0,4\} \tag{3.29}$$

$$\phi_{\mathbf{a_3b_3}}=\{0,0,0,4,8,4,0,0\} \tag{3.30}$$

$$\phi_{\mathbf{a_4b_4}}=\{0,0,0,0,4,8,4,0\} \tag{3.31}$$

So, the sum of cross correlation function is

$$\phi_{\mathbf{a_1b_1}} + \phi_{\mathbf{a_2b_2}} + \phi_{\mathbf{a_3b_3}} + \phi_{\mathbf{a_4b_4}} = \{-8,4,0,4,16,4,8,4\} \tag{3.32}$$

In this case, the sum of periodic cross correlation is not zero.

3.3 Invariance Properties of Periodic Complementary Sequences

By applying the invariance operation of PCS, another PCS is generated [4]. Some of the invariance properties of PCS are shown as under:

1) Negating any number of sequences does not change the PCS property.

2) Reversing any sequence keeps the property unchanged. By reversing, we mean that if $\mathbf{a} = \{a(0)\ a(1)\ ...a(N-1)\}$ be one sequence of length N, then another sequence $\mathbf{b}$ is reversal of sequence $\mathbf{a}$ if

$$b(n) = a(n)\ for\ n = 0\ and\ \ a(N-n)\ elsewhere \tag{3.33}$$

3) Reflection of any number of sequences in set yield another PCS.

A sequence $\mathbf{b}$ is said to be reflection of sequence $\mathbf{a}$ if

b(n)=a(N-1-n)

4) Alternately negating the elements of all the sequences yield another PCS.

5) If we give any amount of cyclic shift to any numer of sequences in the set, PCS property still holds.

3.4 Construction of PCS of Higher Period and Cardinality

If some PCS is available to us, we can generate new Periodic Complementary Sequences with longer period and more number of sequences per set [4]. Computer search can be used to find out the inital PCS to construct higher order PCS. Some of the ways are explained below:

1) One very obvious way to form a S_P^{tN} from S_P^N is to repeat the sequences t times.

2) Consider a $S_P^N(\mathbf{a}_i)$ with an even number of sequences P. Then the mate of this PCS is the set of sequences given by

$$(\tilde{\mathbf{a}}_1, -\tilde{\mathbf{a}}_0, \tilde{\mathbf{a}}_3, -\mathbf{a}_2, ...\tilde{\mathbf{a}}_{P-1}, -\tilde{\mathbf{a}}_{P-2})$$

Here $\tilde{\mathbf{a}}$ denotes the reverse of sequence $\mathbf{a}$.

3) A new PCS can be generated with existing PCS and its mate having double the length of the sequence. Given a $S_P^N(\mathbf{a}_i)$ and its mate $S_P^N(\mathbf{b}_i)$, the set of interleaved sequences

$$\mathbf{c}_i = \mathbf{a}_i \bowtie \mathbf{b}_i \qquad 0 \leq i \leq P-1 \tag{3.34}$$

form a new PCS with length of sequences being equal to 2N. Interleaving means that if ($\mathbf{a}$,$\mathbf{b}$,$\mathbf{c}$)be sequences of length N then

$$\mathbf{d} = \mathbf{a} \bowtie \mathbf{b} \bowtie \mathbf{c} = (a(0)\ b(0)\ c(0)\ \ a(1)\ b(1)\ c(1)\ ...a(N-1)\ b(N-1)\ c(N-1)) \tag{3.35}$$

denotes the interleaved sequence.

4) Let a $S_P^N(\mathbf{a}_i)$ be given and the binary Q*P matrix with orthogonal columns be given. Then a S_Q^{NP} is generated with Q sequences.

First sequence-

$$\mathbf{a}_0.h_{0,0} \bowtie \mathbf{a}_1.h_{0,1} \ldots \bowtie \mathbf{a}_{P-1}.h_{0,P-1}$$

Qth sequence-

$$\mathbf{a}_0.h_{Q-1,0} \bowtie \mathbf{a}_1.h_{Q-1,1} \ldots \bowtie \mathbf{a}_{P-1}.h_{Q-1,P-1}$$

Thus if we have PCS with four sequences and each sequence of length 7, and a 4 by 4 matrix of orthogonal columns, we can generate PCS of length 28.

5) Consider two $S_P^N(\mathbf{a}_i)$ and $S_Q^M(\mathbf{b}_i)$.The lengths N and M and number of sequences P and Q are relatively prime. Then the the products of the elements of the repeated $S_R^L(\tilde{\mathbf{a}}_j)$ by the elements of the repeated $S_R^L(\tilde{\mathbf{b}}_j)$with the same index build one period of a new $S_R^L(\mathbf{c}_i)$ with length L=N.M and number of sequences R=P.Q [6]. Thus if we have $S_3^4(\mathbf{a}_i)$ and $S_4^7(\mathbf{b}_i)$, we can construct $S_{12}^{28}(\mathbf{c}_i)$.

6) Given $S_P^N(\mathbf{a}_i, \mathbf{b}_i, ...\mathbf{f}_i)$ in which any two PCS are mates of each other. Let

$$\mathbf{X}_j = \mathbf{x}_0 \bowtie \mathbf{x}_0, \mathbf{x}_1 \bowtie \mathbf{x}_1, ..., \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1,} - \mathbf{x}_0 \bowtie \mathbf{x}_0, -\mathbf{x}_1 \bowtie \mathbf{x}_1, ..., -\mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1} \tag{3.36}$$

denote a set of 2P interleaved sequences and

$$\mathbf{X}_j^- = -\mathbf{x}_0 \bowtie \mathbf{x}_0, -\mathbf{x}_1 \bowtie \mathbf{x}_1, ... - \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1}, \mathbf{x}_0 \bowtie \mathbf{x}_0, \mathbf{x}_1 \bowtie \mathbf{x}_1, ..., \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1} \tag{3.37}$$

denote another set of 2P interleaved sequences.

denotes the interleaved sequence.

4) Let a $S_P^N(\mathbf{a}_i)$ be given and the binary Q*P matrix with orthogonal columns be given. Then a S_Q^{NP} is generated with Q sequences.

First sequence-

$$\mathbf{a}_0.h_{0,0} \bowtie \mathbf{a}_1.h_{0,1}... \bowtie \mathbf{a}_{P-1}.h_{0,P-1}$$

Qth sequence-

$$\mathbf{a}_0.h_{Q-1,0} \bowtie \mathbf{a}_1.h_{Q-1,1}... \bowtie \mathbf{a}_{P-1}.h_{Q-1,P-1}$$

Thus if we have PCS with four sequences and each sequence of length 7, and a 4 by 4 matrix of orthogonal columns, we can generate PCS of length 28.

5) Consider two $S_P^N(\mathbf{a}_i)$ and $S_Q^M(\mathbf{b}_i)$.The lengths N and M and number of sequences P and Q are relatively prime. Then the the products of the elements of the repeated $S_R^L(\tilde{\mathbf{a}}_j)$ by the elements of the repeated $S_R^L(\tilde{\mathbf{b}}_j)$with the same index build one period of a new $S_R^L(\mathbf{c}_i)$ with length L=N.M and number of sequences R=P.Q [6]. Thus if we have $S_3^4(\mathbf{a}_i)$ and $S_4^7(\mathbf{b}_i)$, we can construct $S_{12}^{28}(\mathbf{c}_i)$.

6) Given $S_P^N(\mathbf{a}_i, \mathbf{b}_i, ...\mathbf{f}_i)$ in which any two PCS are mates of each other. Let

$$\mathbf{X}_j = \mathbf{x}_0 \bowtie \mathbf{x}_0, \mathbf{x}_1 \bowtie \mathbf{x}_1, ..., \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1,} - \mathbf{x}_0 \bowtie \mathbf{x}_0, -\mathbf{x}_1 \bowtie \mathbf{x}_1, ..., -\mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1} \tag{3.36}$$

denote a set of 2P interleaved sequences and

$$\mathbf{X}_j^- = -\mathbf{x}_0 \bowtie \mathbf{x}_0, -\mathbf{x}_1 \bowtie \mathbf{x}_1, ... - \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1}, \mathbf{x}_0 \bowtie \mathbf{x}_0, \mathbf{x}_1 \bowtie \mathbf{x}_1, ..., \mathbf{x}_{P-1} \bowtie \mathbf{x}_{P-1} \tag{3.37}$$

denote another set of 2P interleaved sequences.

Then the set of sequences

$$\{\mathbf{A}_j, \mathbf{B}_j, ..., \mathbf{F}_j, \mathbf{A}_j^-, \mathbf{B}_j^-, ...\mathbf{F}_j^-\} \tag{3.38}$$

built a new S_{2P}^{2N} The method explained above is very useful in doubling the length of sequences as well as the number of sequences. Also new PCS can be generated which will be mates of each other. So, if we have two PCS of length 2 with two sequences per set and both are mates of each other, then by applying above method, four sets of PCS can be generated in which all will be mates of each other. The length of PCS will be 4 with 4 sequences per set.

7) If a $S_P^N(\mathbf{a}_i)$ is generated from a perfect binary array, then the following set of sequences

$$(\mathbf{a}_0 \bowtie \mathbf{a}_1 \bowtie ... \bowtie \mathbf{a}_{P-1}, a_1 \bowtie \mathbf{a}_2 \bowtie ... \bowtie \mathbf{a}_{0,...,}\mathbf{a}_{P-1} \bowtie \mathbf{a}_0 \bowtie ... \bowtie \mathbf{a}_{P-2)} \tag{3.39}$$

constructS_P^{NP}. Existence conditions of higher PCS from initial PCS is given in [6].

3.5 Periodic Complementary Sequences for Multicarrier CDMA

As explained above, PCS is not a single sequence but a set of sequences. The cross correlation between different sequences of mates is not zero but the sum of cross correlations between corresponding sequences of mates is zero everywhere. This useful property can be exploited in multicarrier CDMA system. In [8], Aperiodic Complementary Sequences(ACS) have been used for Multicarrier CDMA in AWGN channel in synchronous and asynchronous conditions. We propose a new scheme, wherein, each

user is assigned a set of PCS. Thus, unlike conventional CDMA where each user is assigned only one signature sequence, here each user is given a set of sequences. So different users are assigned PCS which are mates of each other. A user transmits data on multiple carriers. The signal on each carrier is spread using distinct sequences of a set of PCS. Thus, the proposed CDMA system using PCS is a multicarrier system rather than single carrier direct spread system. The spreading gain of such a system is length of each sequence of PCS multiplied by the number of carriers. The system is quite flexible in the sense that for a given spreading gain N, we can vary the number of carriers used. For example, for a spreading gain of 16, we can have 2 carriers or 4 carriers with length of spreading sequence 8 and 4 respectively for each carrier. (3.40) to (3.43) show four such sets of PCS mates. Each PCS has four sequences and length of each sequence is also four.

$$\mathbf{a} = [++++;++--;-+-+;-++-] \tag{3.40}$$

$$\mathbf{b} = [++--;----;-++-;+-+-] \tag{3.41}$$

$$\mathbf{c} = [-+-+;-++-;++++;++--] \tag{3.42}$$

$$\mathbf{d} = [-++-;+-+-;++--;----] \tag{3.43}$$

For 2 carriers, (3.44) and (3.45) show two sets of PCS mates. Each PCS consists of 2 sequences and length of each sequence is 8.

$$\mathbf{a} = [+++-+---;+--+----] \tag{3.44}$$

$$\mathbf{b} = [+----+--;-+++-+--] \tag{3.45}$$

The number of users accomodated in a system depends on the number of orthogonal

sequence sets which are available. However, for a given number and length of sequences in PCS, there is no general expression which gives the number of PCS mates that exist. They have to be found using extensive computer search algorithms. For a given spreading gain N, number of PCS mates which exist will not be more than N. This is due to the fact that for spreading gain N, we cannot have more than N sequences which are orthogonal to each other for zero shift.

3.5.1 Transmitter Structure

The transmitter of a user is shown in Figure 3.1.

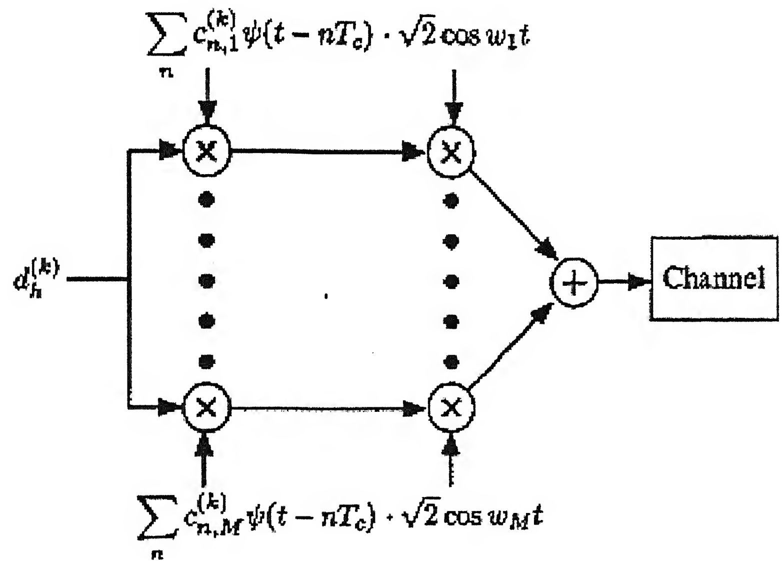

Figure 3.1: Transmitter structure

As the system is a multicarrier system, the incoming data of a user is first converted from serial to parallel. Each branch has the same data bits. After this the data is spread by PCS assigned to that user. By spreading, we mean that data bits on the first branch are multiplied by the first sequence of PCS, second branch by the second sequence of

PCS and so on. After multiplication with PCS, the data is then passed through a pulse shaping filter. The output of wave shaping filters are modulated onto different carriers. In order to avoid intereference between different carriers, the spectra around carriers should be non overlapping. The modulated signal from different branches is then added and transmitted.

Consider a system with K users and each user has a PCS containing M sequences. As each sequence occupies one carrier, we will have a total of M carriers. Let the length of each sequence be N. Let $\{C_i^{(k)}, i = 1, 2, ..., M\}$ be the PCS assigned to user k and $c_{n,i}^{(k)}$ be the nth chip of periodically repeated ith sequence of kth user. Then the transmitted signal of user k is given by:

$$s(t) = \sum_{i=1}^{M} \left[\sum_{n=0}^{\infty} d^{(k)} c_{n,i}^{(k)} \Psi(t - nT_c - \tau_k) \right] cos(\omega_i t) \tag{3.46}$$

where $d^{(k)}$ is the data bit of kth user, τ_k is the delay of kth user, ω_i is the frequency of ith carrier and $\Psi(t)$ is the pulse shaping function.

3.5.2 Receiver

The receiver is shown in Figure 3.2. The received signal corrupted by noise is converted from serial to parallel. The number of parallel branches is equal to the number of frequencies used for transmission. At the front end of the receiver, first of all we separate the different frequency bands from each other. Demodulation is then performed on different branches. We assume that receiver has knowledge of carrier phase and frequency. The demodulated data is then passed through matched filter and operated by PCS of that particular user. For this, we take the periodic correlation of data on

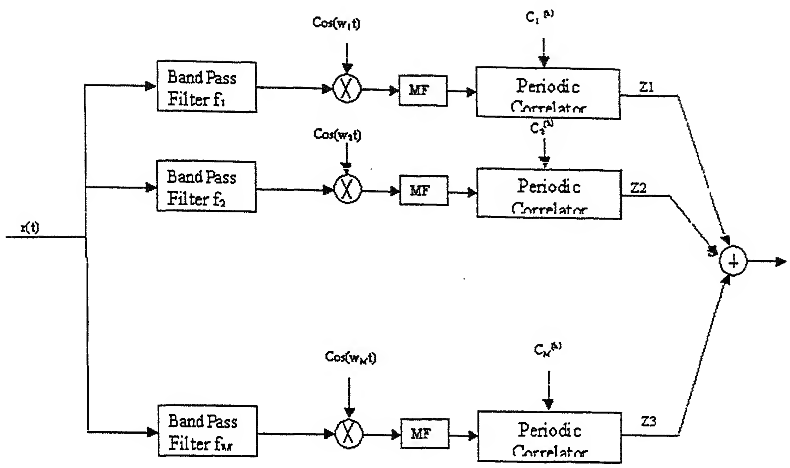

Figure 3.2: Receiver structure

different branches with the sequences of PCS. The periodic correlator for first branch is shown in Figure 3.3. In order to take the periodic correlation, the filtered signal $y^{(k)}$ at the input of periodic decorrelator is divided into 2 parallel streams. On the first stream, we multiply the twice repeated signal $y^{(k)}$ with twice repeated sequence $C_1^{(k)}$. On the second stream, we multiply $y^{(k)}$ with $C_1^{(k)}$. The output signal of second stream is then subtracted from that of first stream.

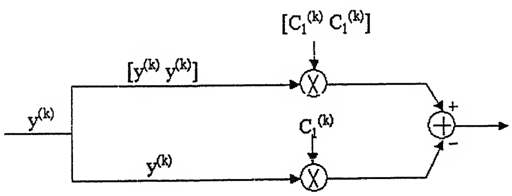

Figure 3.3: Periodic Correlator

The decorrelated signal on different branches is added together and is given to the decision device for detection. Since different users are assigned PCS which are mates of each other and orthogonality is based on addition of periodic cross correlations, the effect of other users is eliminated completely. In other words, we can say that MAI is made nearly equal to zero by utilizing correlation property of Periodic Complementary Sequences. The operation of periodic correlator is explained with an example below:

Let data bit of first (desired) user be +1. Let the PCS assigned to it be [1 1 1 -1; 1 -1 -1 -1]. Let data bit of second (interferer) user be -1. Let the PCS assigned to it be [1 -1 -1 -1; -1 1 -1 -1] which is mate of PCS assigned to first user. Let second user has a delay of 3 chips with respect to first user. Then first user will transmit [1 1 1 -1] on first carrier and [1 -1 -1 -1] on second carrier. Second user will transmit [-1 1 1 1] and [1 -1 1 1] on the two carriers respectively. We will consider MAI only and neglect the effect of noise. At the inputs of periodic correlators, we will get the desired signal with interferer's signal. First, we consider periodic correlator on first carrier frequency. On the first stream of the periodic correlator, [1 1 1 1 1 1 1 -1] will be multiplied by [-1 1 1 1 -1 1 1 1] and the result will be +5. On the second stream, [1 1 1 -1] will be multiplied by [-1 1 1 1] and the result will be +1. Hence, the output of periodic correlator will be difference of two which is 4. Similarly, the output of correlator on second branch will be -4. The sum of outputs of both the correlators is 0 which illustrates that effect of second user is eliminated completely.

In case of ACS, we simply multiply $y^{(k)}$ with $C_1^{(k)}$ and add the outputs of all branches.

3.5.3 Advantages of PCS

This scheme offers several advantages. First, different users are orthogonal to each other in synchronous as well as asynchronous conditions provided the delay between users is an integral multiple of chip duration. Thus, MAI is ideally reduced to zero. Reduction in MAI may result in accomodation of more number of users for the same bit error performance if we have enough sequence sets. Secondly, due to multicarrier transmission, a wideband signal is converted into several narrowband signals. Thus the effect of frequency selective fading is less. Also since the autocorrelation sidelobes cancel out completely, synchronization is easier.

Chapter 4

Simulation Results and Conclusions

In this chapter, we have shown simulation results for multicarrier CDMA system using PCS in AWGN and slow flat Rayleigh fading channel. For multicarrier CDMA system employing PCS, we have used S_4^8 i.e. each user is assigned a set of 4 sequences and each sequence is of length 8. Thus, the multicarrier system is a 4 carrier system. The performance of multicarrier system using PCS is compared with DS CDMA system. Gold sequences and MLSR sequences of length 31 are used for DS CDMA. Thus, the spreading gains of both the systems are nearly equal (31 for DS CDMA and 32 for multicarrier). Simulation is carried out by transmitting 1000 randomly generated bits during one run of the program. The program is run 100 times and each time the bits are generated randomly. So, we are effectively transmitting 100000 bits. For asynchronous system, around 15 delay values have been taken for 2 user case and 35 delay combinations have been taken for 3 users. The Bit Error Rate is then calculated by averaging out the outputs. Both the multicarrier and direct spread CDMA systems occupy same bandwidth. BPSK modulation is used at the transmitter. It is assumed

that modulation and demodulation are perfectly coherent and phase errors do not occur between transmitter and receiver.

4.1 Spreading Sequences Used in Simulations

The following Gold sequences have been used for simulations:

$$c1 = [-+-++-+++---+-----+--+---++---+]$$

$$c = [-+++----+----++--+--+-++++-----]$$

$$c2 = [----++-++--+-+--+++++-++--+--++]$$

Here, user having spreading sequence **c** is desired user.

Following are the 3 MLSR sequences used for simulations:

$$c1 = [---+-+-++-+-----+--+--+++++-+++]$$

$$c = [----+--+-++--+++++---++-+++-+-+]$$

$$c2 = [--++-+++++-+---+--+-+-++----+++]$$

The four set of PCS which are assigned to different users are

$$b = [++++++--;++------;-+-+-++-;-++-+-+-]$$

$$bx = [-+-+-++-;-++-+-+-;++++++--;++------]$$

$$bxx = [++------;--++----;-++-+-+-;+--++-+-]$$

$$bxxx = [-++-+-+-;+--++-+-;++------;--++----]$$

Here, user which is assigned PCS **b** is desired user whose data bits we have to detect. It is interesting to note that in the above set of PCS, each PCS is a mate of every other

PCS. This property is responsible for the signals of different users to be orthogonal even in an asynchronous system. For a spreading gain of 32, it is also possible to use either 2 ,4 or 8 carriers. For transmitting data over 2 and 8 carriers, we require S_2^{16} and S_8^4 respectively. **a** and **b** are examples of two PCS mates for transmitting data over 2 carriers.

$$\begin{aligned}
\mathbf{a} &= [+++-+---+--+----;-++++-++-+++-+--] \\
\mathbf{b} &= [---+-+++-++-++++;-++++-++-+++-+--]
\end{aligned}$$

For transmitting data over 8 carriers, following sets of PCS mates can be used:

$$\begin{aligned}
\mathbf{a} &= [++++;++--;--++;----;-+-+;-++-;+--+;+-+-] \\
\mathbf{b} &= [--++;----;++++;++--;+--+;+-+-;-+-+;-++-] \\
\mathbf{c} &= [-+-+;-++-;+--+;+-+-;++++;++--;--++;----] \\
\mathbf{d} &= [+--+;+-+-;-+-+;-++-;--++;----;++++;++--]
\end{aligned}$$

It is not possible to use 16 carriers. If 16 carriers are used, the length of each spreading sequence will be 2. For transmitting data over 16 carriers we require 16 sequences of length 2 but maximum number of sequences of length 2 is only 4.

4.2 Channel

Two types of channel models are used for simulations.

First is the AWGN channel . In AWGN channel, the received signal is given by **r**=**t**+**n**. Here **r** is received signal, **t** is transmitted signal and **n** is additive white gaussian noise

with zero mean and power spectral density $N_0/2$. The variance of noise is assumed to be σ^2. If the spreading gain of CDMA system is N, then following relation holds

$$E_b/N_0 = N.SNR \tag{4.1}$$

Here, E_b is energy per bit.

SNR is signal to noise ratio.

If power of a user is unity, then

$$E_b/N_0 = N/\sigma^2 \tag{4.2}$$

Secondly, we simulated the system performance in slow flat Rayleigh fading channel. In this case, the received sequence is given by $\mathbf{r} = \alpha\mathbf{t}$. Here, α is independent, identically distributed Rayleigh random variable with second moment equal to unity. By slow fading, we mean that channel impulse remains constant during one bit period. In both AWGN and Rayleigh fading channel, the power of users is set to unity.

4.3 Simulation Results

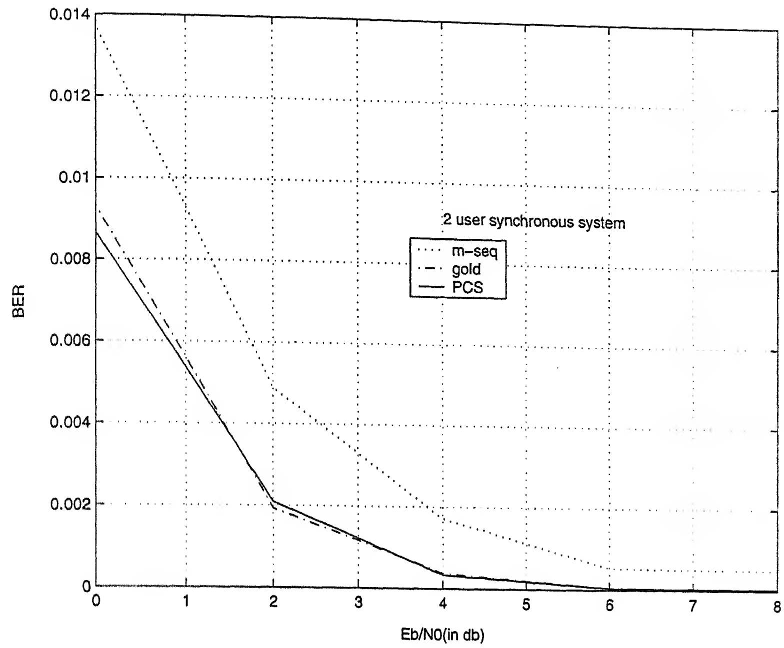

Figure 4.1: PCS compared with Gold and MLSR sequences in synchronous system in AWGN channel. Number of users is 2.

Figure 4.1 shows the performance of Multicarrier CDMA using PCS. The Bit Error Rate of PCS is compared with DS CDMA system employing Gold and MLSR sequences for different values of E_b/N_0. Both the user's transmissions are synchronous. It is clear from the Figure that BER acheived using PCS is almost same as obtained using Gold sequence. However, improvement is obtained as compared to MLSR sequences. This is due to the fact that both Gold and PCS exhibit nearly same cross correlation at zero shift and hence there is not much difference.

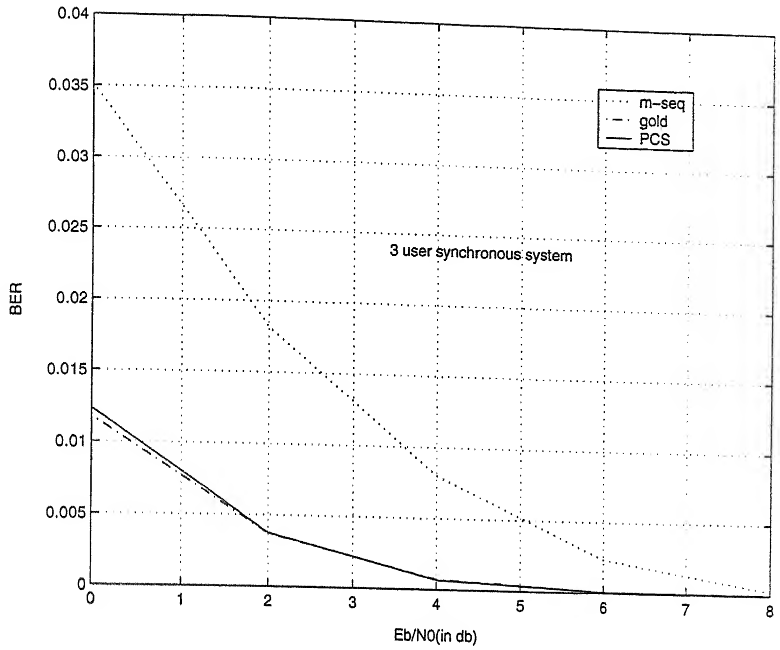

Figure 4.2: PCS compared with Gold and MLSR sequences in synchronous system in AWGN channel. Number of users is 3.

In Figure 4.2 the performance of Multicarrier CDMA using PCS is compared with DS CDMA system employing Gold and MLSR sequences. 3 users are accomodated in the system and system is synchronous. As earlier, it is clear from this Figure also that Bit Error Rate acheived using PCS is almost same as obtained using Gold sequence in synchronous system. However, improvement is obtained as compared to MLSR sequences.

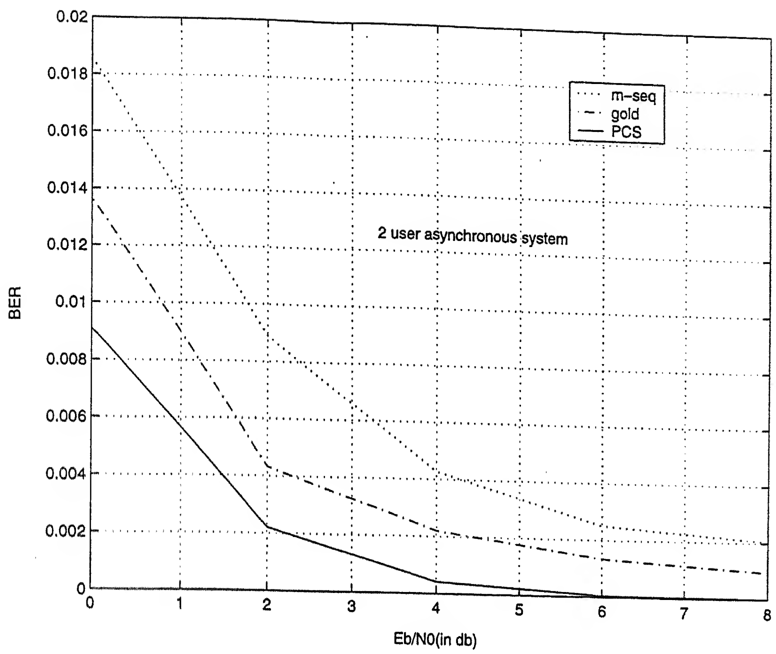

Figure 4.3: PCS compared with Gold and MLSR sequences in asynchronous system in AWGN channel.2 users are considered .

Figure 4.3 brings out the comparison between PCS and Gold and MLSR sequences in AWGN channel when the transmission is asynchronous and 2 users co-exist in the system. One thing, that is very clear, from the figure is that Multicarrier CDMA using PCS outperforms DS CDMA system using Gold and MLSR sequences in asynchronous mode. This is due to the excellent cross correlation property of PCS mates. The cross correlation of PCS mates is zero at all shifts. Thus a significant gain of about 2.5 db is acheived over conventional Gold and MLSR sequences. The BER performance has been obtained by delaying the transmission of second user by arbitrary amount(multiple of chips) and the final result is obtained by taking average.

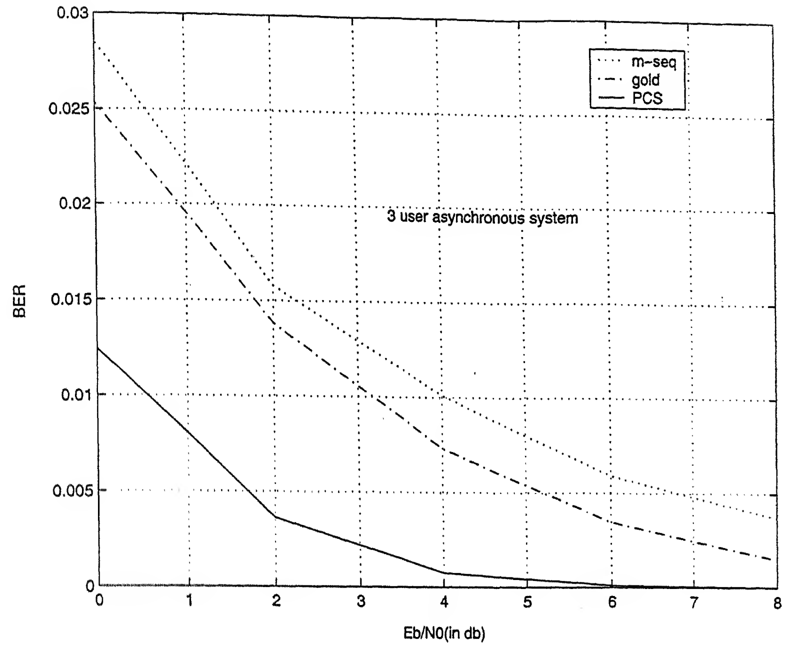

Figure 4.4: PCS compared with Gold and MLSR sequences in asynchronous in AWGN channel. 3 users are considered .

Figure 4.4 supports the results obtained in Figure 4.3. There are 3 users in the system and mode of transmission is asynchronous. It is quite obvious from the figure that the BER performance of PCS is very good than Gold and MLSR sequences. Gain of about 3 db is obtained. The reason is explained earlier. The orthogonality among either Gold sequences or MLSR sequences is destroyed by the asynchronous bit streams from different users while it is preserved in case of PCS.

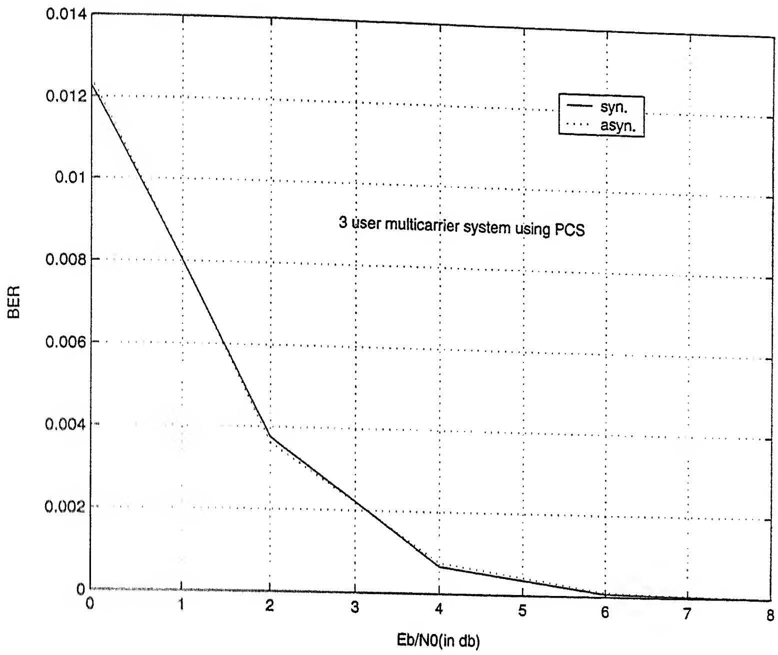

Figure 4.5: Bit Error Rate Vs Eb/N0 for synchronous and asynchronous system using PCS in AWGN channel. 3 users are considered .

Figure 4.5 demonstrates that error rate obtained using PCS is almost same whether it is synchronous system or asynchronous system. The two curves are nearly on top of each other. The reason is that periodic cross correlation values of PCS mates is 0 at each and every shift including the zero shift. The simulation results obtained in above figure clearly highlight this property.

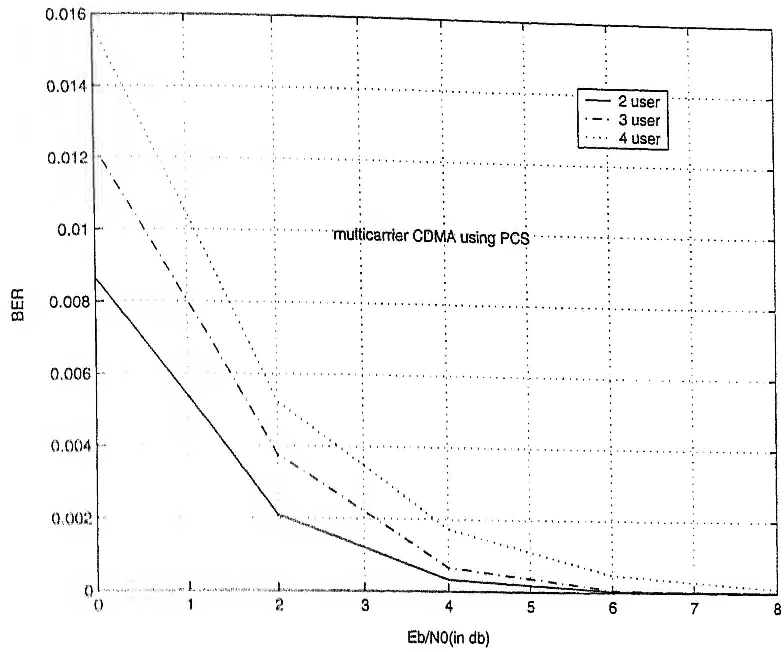

Figure 4.6: Comparison of Bit Error Rate Vs Eb/N0 using PCS in AWGN channel for different number of users in asynchronous system.

Figure 4.6 shows the MAI independent property of PCS. The Multicarrier system using PCS is simulated for 2 users, 3 users and 4 users in AWGN channel. Here, users are asynchronous. We can see that there is not much difference between the error rates obtained for different number of users. Almost identical BER performance is achieved irrespective of the number of users. This is not possible in traditional sequences used in an asynchronous CDMA system.

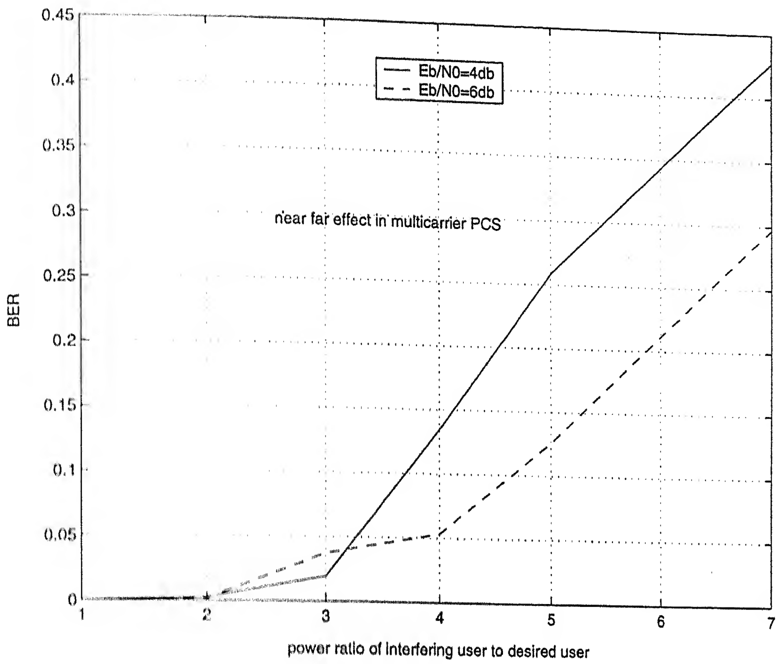

Figure 4.7: Demonstration of near far effect using PCS in AWGN channel. Noise power is kept constant and interfering user power is increased.

In Figure 4.7, we have tried to show the near far effect on the performance of a PCS Multicarrier system. Here, power of interfering user is increased compared to desired user's power. Simulations have been carried out for $E_b/N_0 = 4$ and $E_b/N_0 = 6$.

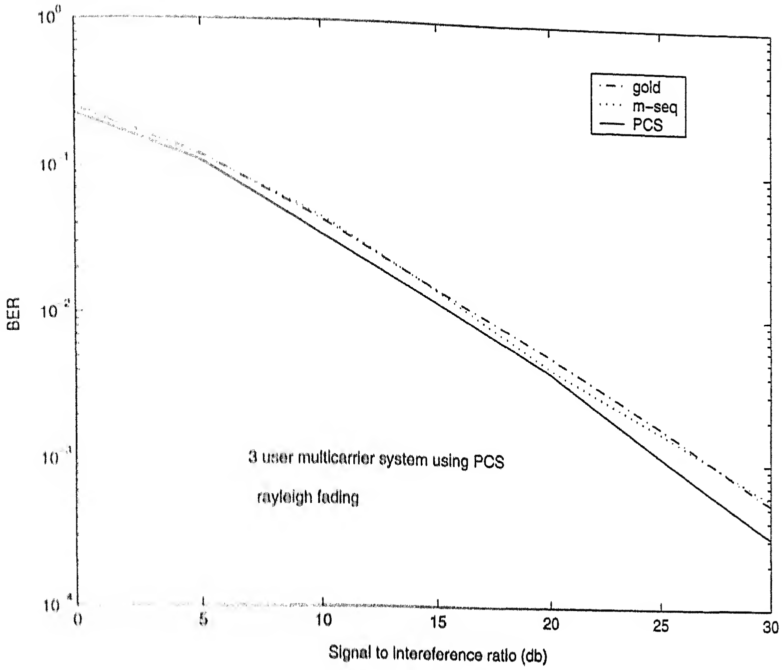

Figure 4.8: PCS compared with Gold and MLSR sequences in slow flat Rayleigh fading channel. Number of users is 3.

Figure 4.8 brings out the comparison between PCS and Gold and MLSR sequences in case of slow flat Rayleigh fading channel.Signal to Intereference Ratio is the ratio of desired user's power and sum of powers of both interfering users. White noise is not taken in account. The fading is assumed to be slow and flat. 3 users are in the system. From the figure, it is seen that PCS system offers no advantage in case of fading. This is due to the fact that unique orthogonality poperty of PCS mates is destroyed by fading. So, there is no improvement in Multicarrier CDMA involving PCS compared to DS CDMA using Gold and MLSR sequences.

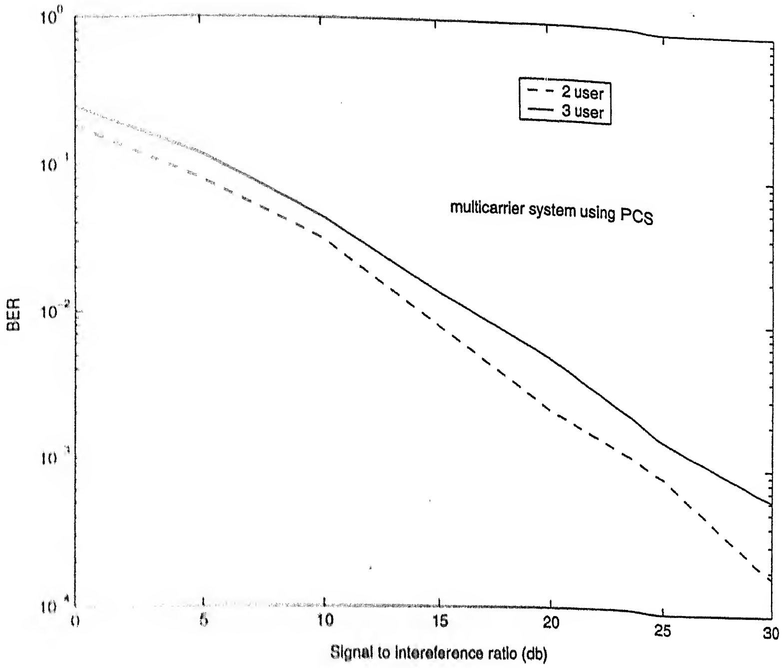

Figure 4.9: Comparison of Bit Error Rate Vs Eb/N0 using PCS in slow flat Rayleigh fading channel for different number of users. For comparison, 2 users and 3 users are taken.

Figure 4.9 compares the PCS system for 2 users and 3 users in slow flat Rayleigh fading channel in asynchronous conditions. The curves are quite obvious. Unlike the AWGN channel where there was little effect of the number of users on BER (Figure 4.6), here the BER for 2 users is less than that for 3 users. These results confirm again that in case of fading channels, orthogonality property of PCS no longer holds good and they behave like traditional sequences.

4.4 Conclusions

In this work, we have compared the performance of Multicarrier system using PCS with DS CDMA system using Gold and MLSR sequences. The following conclusions can be easily drawn:

1) Multicarrier system employing PCS outperforms the DS CDMA system with Gold and MLSR sequences in asynchronous system in case of AWGN channel.

2) The BER performance of PCS is almost same in synchronous and asynchronous system in AWGN channel.

3) Below a certain limit, number of users do not effect the BER of PCS in asynchronous system in AWGN channel.

4) PCS offers no advantage over Gold or MLSR sequences in case of fading channel.

5) Whereas ACS exist for only even number of sequences, PCS exist for both odd as well as even number of sequences. Hence, PCS can also be used in Multicarrier CDMA with odd number of sequences.

6) For a given spreading gain, number of carriers used in Multicarrier CDMA system using PCS can be varied.

4.5 Scope For Future Work

The sequences of a set of PCS can be concatenated to form a single sequence . The concatenated sequences thus obtained will be orthogonal to each other in synchronous conditions. The performance of these sequences can be compared with conventional sequences used in DS CDMA in asynchronous environment. There is no general ex-

pression which gives number of PCS mates for a given length and number of sequences in a PCS set. It may be worthwhile to explore these issues so that we can have a better idea of the capacity limit of a PCS based CDMA system.

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